Package 'sandwich'

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Title Robust Covariance Matrix Estimators

Description Object-oriented software for model-robust covariance matrix estimators. Starting out from the basic robust Eicker-Huber-White sandwich covariance methods include: heteroscedasticity-consistent (HC) covariances for cross-section data; heteroscedasticity- and autocorrelation-consistent (HAC) covariances for time series data (such as Andrews' kernel HAC, Newey-West, and WEAVE estimators); clustered covariances (one-way and multi-way); panel and panel-corrected covariances; outer-product-of-gradients covariances; and (clustered) bootstrap covariances. All methods are applicable to (generalized) linear model objects fitted by lm() and glm() but can also be adapted to other classes through S3 methods. Details can be found in Zeileis et al. (2020) <doi:10.18637/jss.v095.i01>, Zeileis (2004) <doi:10.18637/jss.v011.i10> and Zeileis (2006) <doi:10.18637/jss.v016.i09>.

Depends R (>= 3.0.0)

Imports stats, utils, zoo

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`bread` *Bread for Sandwiches*

Description

Generic function for extracting an estimator for the bread of sandwiches.

Usage

```
bread(x, ...)
```

Arguments

`x`	a fitted model object.
`...`	arguments passed to methods.

Value

A matrix containing an estimator for the expectation of the negative derivative of the estimating functions, usually the Hessian. Typically, this should be an $k \times k$ matrix corresponding to k parameters. The rows and columns should be named as in `coef` or `terms`, respectively.

The default method tries to extract `vcov` and `nobs` and simply computes their product.

References

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

Zeileis A, Köll S, Graham N (2020). "Various Versatile Variances: An Object-Oriented Implementation of Clustered Covariances in R." *Journal of Statistical Software*, **95**(1), 1–36. doi:10.18637/jss.v095.i01

See Also

lm, glm

Examples

```
## linear regression
x <- sin(1:10)
y <- rnorm(10)
fm <- lm(y ~ x)

## bread: n * (x'x)^{-1}
bread(fm)
solve(crossprod(cbind(1, x))) * 10
```

---

estfun *Extract Empirical Estimating Functions*

Description

Generic function for extracting the empirical estimating functions of a fitted model.

Usage

```
estfun(x, ...)
```

Arguments

x	a fitted model object.
...	arguments passed to methods.

Value

A matrix containing the empirical estimating functions. Typically, this should be an $n \times k$ matrix corresponding to n observations and k parameters. The columns should be named as in coef or terms, respectively.

The estimating function (or score function) for a model is the derivative of the objective function with respect to the parameter vector. The empirical estimating functions is the evaluation of the estimating function at the observed data (n observations) and the estimated parameters (of dimension k).

References

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

Zeileis A, Köll S, Graham N (2020). "Various Versatile Variances: An Object-Oriented Implementation of Clustered Covariances in R." *Journal of Statistical Software*, **95**(1), 1–36. doi:10.18637/jss.v095.i01

See Also

`lm`, `glm`

Examples

```
## linear regression
x <- 1:9
y <- sin(1:9/5)
m <- lm(y ~ x)

## estimating function: (y - x'beta) * x
estfun(m)
residuals(m) * cbind(1, x)
```

`InstInnovation` *Innovation and Institutional Ownership*

Description

Firm-level panel data on innovation and institutional ownership from 1991 to 1999 over 803 firms. The observations refer to different firms over different years.

Usage

```
data("InstInnovation")
```

Format

A data frame containing 6208 observations on 25 variables.

company factor. Company names.

sales numeric. Sales (in millions of dollars).

acompetition numeric. Constant inverse Lerner index.

competition numeric. Varying inverse Lerner index.

capital numeric. Net stock of property, plant, and equipment.

cites integer. Future cite-weighted patents.

precites numeric. Presample average of cite-weighted patents.

dprecites factor. Indicates zero precites.

patents integer. Granted patents.

drandd factor. Indicates a zero R&D stock.

randd numeric. R&D stock (in millions of dollars).

employment numeric. Employment (in 1000s).

sp500 factor. Membership of firms in the S&P500 index.

tobinq numeric. Tobin's q.

value numeric. Stock market value.

institutions numeric. Proportion of stock owned by institutions.

industry factor. Four-digit industry code.

year factor. Estimation period.

top1 numeric. Share of the largest institution.

quasiindexed numeric. Share of "quasi-indexed" institutional owners.

nonquasiindexed numeric. Share of "non-quasi-indexed" institutional owners.

transient numeric. Share of "transient" institutional owners.

dedicated numeric. Share of "dedicated" institutional owners.

competition4 numeric. Varying inverse Lerner index in the firm's four-digit industry.

subsample factor. Subsample for the replication of columns 1–5 from Table 4 in Aghion et al. (2013).

Details

Aghion et al. (2013) combine several firm level panel datasets (e.g., USPTO, SEC and Compustat) to examine the role of institutional investors in the governance of innovation. Their baseline to model innovation is the Poisson model, but they also consider negative binomial models. Berger et al. (2017) argue that nonlinearities in the innovation process emerge in case that the first innovation is especially hard to obtain in comparison to succeeding innovations. Then, hurdle models offer a useful way that allows for a distinction between these two processes. Berger et al. (2017) show that an extended analysis with negative binomial hurdle models differs materially from the outcomes of the single-equation Poisson approach of Aghion et al. (2013).

Institutional ownership (institutions) is defined as the proportion of stock owney by institutions. According to Aghion et al. (2013), an institutional owner is defined as an institution that files a Form 13-F with the Securities and Exchange Commission (SEC).

Future cite-weighted patents (cites) are used as a proxy for innovation. They are calculated using ultimately granted patent, dated by year of application, and weight these by future citations through 2002 (see Aghion et al. (2013)).

The presample average of cite-weighted patents (precites) is used by Aghion et al. (2013) as a proxy for unobserved heterogeneity, employing the "presample mean scaling" method of Blundell et al. (1999).

The inverse Lerner index in the firm's three-digit industry is used as a time-varying measure for product market competition (competition), where the Lerner is calculated as the median gross margin from the entire Compustat database in the firm's three-digit industry (see Aghion et al. (2013)). A time-invariant measure for competition (acompetition) is constructed by averaging the Lerner over the sample period.

The classification of institutions into "quasiindexed", "transient" and "dedicated" follows Bushee (1998) and distinguishes between institutional investors based on their type of investing. Quasi-indexed institutions are do not trade much and are widely diversified, dedicated institution do not trade much and have more concentrated holdings, and transient institutions often trade and have diversified holdings (see Aghion et al. (2013) and Bushee (1998)).

Source

Data and online appendix of Aghion et al. (2013).

References

Aghion P, Van Reenen J, Zingales L (2013). "Innovation and Institutional Ownership." *The American Economic Review*, **103**(1), 277–304. doi:10.1257/aer.103.1.277

Berger S, Stocker H, Zeileis A (2017). "Innovation and Institutional Ownership Revisited: An Empirical Investigation with Count Data Models." *Empirical Economics*, **52**(4), 1675–1688. doi:10.1007/s0018101611180

Blundell R, Griffith R, Van Reenen J (1999). "Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms." *Review of Economic Studies*, 66(3), 529–554.

Bushee B (1998). "The Influence of Institutional Investors on Myopic R&D Investment Behavior." *Accounting Review*, **73**(3), 655–679.

Examples

```
## Poisson models from Table I in Aghion et al. (2013)

## load data set
data("InstInnovation", package = "sandwich")

## log-scale variable
InstInnovation$lograndd <- log(InstInnovation$randd)
InstInnovation$lograndd[InstInnovation$lograndd == -Inf] <- 0

## regression formulas
f1 <- cites ~ institutions + log(capital/employment) + log(sales) + industry + year
f2 <- cites ~ institutions + log(capital/employment) + log(sales) +
  industry + year + lograndd + drandd
f3 <- cites ~ institutions + log(capital/employment) + log(sales) +
  industry + year + lograndd + drandd + dprecites + log(precites)

## Poisson models
tab_I_3_pois <- glm(f1, data = InstInnovation, family = poisson)
tab_I_4_pois <- glm(f2, data = InstInnovation, family = poisson)
tab_I_5_pois <- glm(f3, data = InstInnovation, family = poisson)

## one-way clustered covariances
vCL_I_3 <- vcovCL(tab_I_3_pois, cluster = ~ company)
vCL_I_4 <- vcovCL(tab_I_4_pois, cluster = ~ company)
vCL_I_5 <- vcovCL(tab_I_5_pois, cluster = ~ company)
```

```
## replication of columns 3 to 5 from Table I in Aghion et al. (2013)
cbind(coef(tab_I_3_pois), sqrt(diag(vCL_I_3)))[2:4, ]
cbind(coef(tab_I_4_pois), sqrt(diag(vCL_I_4)))[c(2:4, 148), ]
cbind(coef(tab_I_5_pois), sqrt(diag(vCL_I_5)))[c(2:4, 148), ]
```

`Investment` *US Investment Data*

Description

US data for fitting an investment equation.

Usage

```
data(Investment)
```

Format

An annual time series from 1963 to 1982 with 7 variables.

GNP nominal gross national product (in billion USD),

Investment nominal gross private domestic investment (in billion USD),

Price price index, implicit price deflator for GNP,

Interest interest rate, average yearly discount rate charged by the New York Federal Reserve Bank,

RealGNP real GNP (= GNP/Price),

RealInv real investment (= Investment/Price),

RealInt approximation to the real interest rate (= Interest - 100 * diff(Price)/Price).

Source

Table 15.1 in Greene (1993)

References

Greene W.H. (1993). *Econometric Analysis*, 2nd edition. Macmillan Publishing Company, New York.

Executive Office of the President (1984). *Economic Report of the President*. US Government Printing Office, Washington, DC.

Examples

```
## Willam H. Greene, Econometric Analysis, 2nd Ed.
## Chapter 15
## load data set, p. 411, Table 15.1
data(Investment)

## fit linear model, p. 412, Table 15.2
fm <- lm(RealInv ~ RealGNP + RealInt, data = Investment)
summary(fm)

## visualize residuals, p. 412, Figure 15.1
plot(ts(residuals(fm), start = 1964),
  type = "b", pch = 19, ylim = c(-35, 35), ylab = "Residuals")
sigma <- sqrt(sum(residuals(fm)^2)/fm$df.residual) ## maybe used df = 26 instead of 16 ??
abline(h = c(-2, 0, 2) * sigma, lty = 2)

if(require(lmtest)) {
## Newey-West covariances, Example 15.3
coeftest(fm, vcov = NeweyWest(fm, lag = 4))
## Note, that the following is equivalent:
coeftest(fm, vcov = kernHAC(fm, kernel = "Bartlett", bw = 5, prewhite = FALSE, adjust = FALSE))

## Durbin-Watson test, p. 424, Example 15.4
dwtest(fm)

## Breusch-Godfrey test, p. 427, Example 15.6
bgtest(fm, order = 4)
}

## visualize fitted series
plot(Investment[, "RealInv"], type = "b", pch = 19, ylab = "Real investment")
lines(ts(fitted(fm), start = 1964), col = 4)


## 3-d visualization of fitted model
if(require(scatterplot3d)) {
s3d <- scatterplot3d(Investment[,c(5,7,6)],
  type = "b", angle = 65, scale.y = 1, pch = 16)
s3d$plane3d(fm, lty.box = "solid", col = 4)
}
```

`isoacf` *Isotonic Autocorrelation Function*

Description

Autocorrelation function (forced to be decreasing by isotonic regression).

Usage

```
isoacf(x, lagmax = NULL, weave1 = FALSE)
```

Arguments

x	numeric vector.
lagmax	numeric. The maximal lag of the autocorrelations.
weave1	logical. If set to TRUE isoacf uses the acf.R and pava.blocks function from the original weave package, otherwise R's own acf and isoreg functions are used.

Details

isoacf computes the autocorrelation function (ACF) of x enforcing the ACF to be decreasing by isotonic regression. See also Robertson et al. (1988).

Value

isoacf returns a numeric vector containing the ACF.

References

Lumley T & Heagerty P (1999). "Weighted Empirical Adaptive Variance Estimators for Correlated Data Regression." *Journal of the Royal Statistical Society B*, **61**, 459–477.

Robertson T, Wright FT, Dykstra RL (1988). *Order Restricted Statistical Inference*. John Wiley and Sons, New York.

See Also

weave, weightsLumley

Examples

```
set.seed(1)
x <- filter(rnorm(100), 0.9, "recursive")
isoacf(x)
acf(x, plot = FALSE)$acf
```

---

kweights *Kernel Weights*

Description

Kernel weights for kernel-based heteroscedasticity and autocorrelation consistent (HAC) covariance matrix estimators as introduced by Andrews (1991).

Usage

```
kweights(x, kernel = c("Truncated", "Bartlett", "Parzen",
  "Tukey-Hanning", "Quadratic Spectral"), normalize = FALSE)
```

Arguments

x	numeric.
kernel	a character specifying the kernel used. All kernels used are described in Andrews (1991).
normalize	logical. If set to TRUE the kernels are normalized as described in Andrews (1991).

Value

Value of the kernel function at x.

References

Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, **59**, 817–858.

See Also

kernHAC, weightsAndrews

Examples

```
curve(kweights(x, kernel = "Quadratic", normalize = TRUE),
      from = 0, to = 3.2, xlab = "x", ylab = "k(x)")
curve(kweights(x, kernel = "Bartlett", normalize = TRUE),
      from = 0, to = 3.2, col = 2, add = TRUE)
curve(kweights(x, kernel = "Parzen", normalize = TRUE),
      from = 0, to = 3.2, col = 3, add = TRUE)
curve(kweights(x, kernel = "Tukey", normalize = TRUE),
      from = 0, to = 3.2, col = 4, add = TRUE)
curve(kweights(x, kernel = "Truncated", normalize = TRUE),
      from = 0, to = 3.2, col = 5, add = TRUE)
```

---

lrvar *Long-Run Variance of the Mean*

Description

Convenience function for computing the long-run variance (matrix) of a (possibly multivariate) series of observations.

Usage

```
lrvar(x, type = c("Andrews", "Newey-West"), prewhite = TRUE, adjust = TRUE, ...)
```

Arguments

x	numeric vector, matrix, or time series.
type	character specifying the type of estimator, i.e., whether kernHAC for the Andrews quadratic spectral kernel HAC estimator is used or NeweyWest for the Newey-West Bartlett HAC estimator.
prewhite	logical or integer. Should the series be prewhitened? Passed to kernHAC or NeweyWest.
adjust	logical. Should a finite sample adjustment be made? Passed to kernHAC or NeweyWest.
...	further arguments passed on to kernHAC or NeweyWest.

Details

lrvar is a simple wrapper function for computing the long-run variance (matrix) of a (possibly multivariate) series x. First, this simply fits a linear regression model x ~ 1 by lm. Second, the corresponding variance of the mean(s) is estimated either by kernHAC (Andrews quadratic spectral kernel HAC estimator) or by NeweyWest (Newey-West Bartlett HAC estimator).

Value

For a univariate series x a scalar variance is computed. For a multivariate series x the covariance matrix is computed.

See Also

kernHAC, NeweyWest, vcovHAC

Examples

```
suppressWarnings(RNGversion("3.5.0"))
set.seed(1)
## iid series (with variance of mean 1/n)
## and Andrews kernel HAC (with prewhitening)
x <- rnorm(100)
lrvar(x)

## analogous multivariate case with Newey-West estimator (without prewhitening)
y <- matrix(rnorm(200), ncol = 2)
lrvar(y, type = "Newey-West", prewhite = FALSE)

## AR(1) series with autocorrelation 0.9
z <- filter(rnorm(100), 0.9, method = "recursive")
lrvar(z)
```

`meat` *A Simple Meat Matrix Estimator*

Description

Estimating the variance of the estimating functions of a regression model by cross products of the empirical estimating functions.

Usage

```
meat(x, adjust = FALSE, ...)
```

Arguments

`x`	a fitted model object.
`adjust`	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k the number of estimated parameters.
`...`	arguments passed to the `estfun` function.

Details

For some theoretical background along with implementation details see Zeileis (2006).

Value

A $k \times k$ matrix corresponding containing the scaled cross products of the empirical estimating functions.

References

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

Zeileis A, Köll S, Graham N (2020). "Various Versatile Variances: An Object-Oriented Implementation of Clustered Covariances in R." *Journal of Statistical Software*, **95**(1), 1–36. doi:10.18637/jss.v095.i01

See Also

`sandwich`, `bread`, `estfun`

Examples

```
x <- sin(1:10)
y <- rnorm(10)
fm <- lm(y ~ x)

meat(fm)
meatHC(fm, type = "HC")
meatHAC(fm)
```

NeweyWest *Newey-West HAC Covariance Matrix Estimation*

Description

A set of functions implementing the Newey & West (1987, 1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix estimators.

Usage

```
NeweyWest(x, lag = NULL, order.by = NULL, prewhite = TRUE, adjust = FALSE,
  diagnostics = FALSE, sandwich = TRUE, ar.method = "ols", data = list(),
  verbose = FALSE)

bwNeweyWest(x, order.by = NULL, kernel = c("Bartlett", "Parzen",
  "Quadratic Spectral", "Truncated", "Tukey-Hanning"), weights = NULL,
  prewhite = 1, ar.method = "ols", data = list(), ...)
```

Arguments

x	a fitted model object. For bwNeweyWest it can also be a score matrix (as returned by estfun) directly.
lag	integer specifying the maximum lag with positive weight for the Newey-West estimator. If set to NULL floor(bwNeweyWest(x, ...)) is used.
order.by	Either a vector z or a formula with a single explanatory variable like ~ z. The observations in the model are ordered by the size of z. If set to NULL (the default) the observations are assumed to be ordered (e.g., a time series).
prewhite	logical or integer. Should the estimating functions be prewhitened? If TRUE or greater than 0 a VAR model of order as.integer(prewhite) is fitted via ar with method "ols" and demean = FALSE. The default is to use VAR(1) prewhitening.
kernel	a character specifying the kernel used. All kernels used are described in Andrews (1991). bwNeweyWest can only compute bandwidths for "Bartlett", "Parzen" and "Quadratic Spectral".
adjust	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k the number of estimated parameters.

diagnostics	logical. Should additional model diagnostics be returned? See vcovHAC for details.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the middle matrix is returned.
ar.method	character. The method argument passed to ar for prewhitening (only, not for bandwidth selection).
data	an optional data frame containing the variables in the order.by model. By default the variables are taken from the environment which the function is called from.
verbose	logical. Should the lag truncation parameter used be printed?
weights	numeric. A vector of weights used for weighting the estimated coefficients of the approximation model (as specified by approx). By default all weights are 1 except that for the intercept term (if there is more than one variable).
...	currently not used.

Details

NeweyWest is a convenience interface to vcovHAC using Bartlett kernel weights as described in Newey & West (1987, 1994). The automatic bandwidth selection procedure described in Newey & West (1994) is used as the default and can also be supplied to kernHAC for the Parzen and quadratic spectral kernel. It is implemented in bwNeweyWest which does not truncate its results - if the results for the Parzen and Bartlett kernels should be truncated, this has to be applied afterwards. For Bartlett weights this is implemented in NeweyWest.

To obtain the estimator described in Newey & West (1987), prewhitening has to be suppressed.

Value

NeweyWest returns the same type of object as vcovHAC which is typically just the covariance matrix.

bwNeweyWest returns the selected bandwidth parameter.

References

Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, **59**, 817–858.

Newey WK & West KD (1987). "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, **55**, 703–708.

Newey WK & West KD (1994). "Automatic Lag Selection in Covariance Matrix Estimation." *Review of Economic Studies*, **61**, 631–653.

Zeileis A (2004). "Econometric Computing with HC and HAC Covariance Matrix Estimators." *Journal of Statistical Software*, **11**(10), 1–17. doi:10.18637/jss.v011.i10

See Also

vcovHAC, weightsAndrews, kernHAC

Examples

```
## fit investment equation
data(Investment)
fm <- lm(RealInv ~ RealGNP + RealInt, data = Investment)

## Newey & West (1994) compute this type of estimator
NeweyWest(fm)

## The Newey & West (1987) estimator requires specification
## of the lag and suppression of prewhitening
NeweyWest(fm, lag = 4, prewhite = FALSE)

## bwNeweyWest() can also be passed to kernHAC(), e.g.
## for the quadratic spectral kernel
kernHAC(fm, bw = bwNeweyWest)
```

PetersenCL *Petersen's Simulated Data for Assessing Clustered Standard Errors*

Description

Artificial balanced panel data set from Petersen (2009) for illustrating and benchmarking clustered standard errors.

Usage

```
data("PetersenCL")
```

Format

A data frame containing 5000 observations on 4 variables.

firm integer. Firm identifier (500 firms).

year integer. Time variable (10 years per firm).

x numeric. Independent regressor variable.

y numeric. Dependent response variable.

Details

This simulated data set was created to illustrate and benchmark clustered standard errors. The residual and the regressor variable both contain a firm effect, but no year effect. Thus, standard errors clustered by firm are different from the OLS standard errors and similarly double-clustered standard errors (by firm and year) are different from the standard errors clustered by year.

Source

https://www.kellogg.northwestern.edu/faculty/petersen/htm/papers/se/test_data.htm

References

Petersen MA (2009). "Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches", *The Review of Financial Studies*, **22**(1), 435–480. doi:10.1093/rfs/hhn053

`PublicSchools` *US Expenditures for Public Schools*

Description

Per capita expenditure on public schools and per capita income by state in 1979.

Usage

```
data("PublicSchools")
```

Format

A data frame containing 51 observations of 2 variables.

Expenditure per capita expenditure on public schools,

Income per capita income.

Source

Table 14.1 in Greene (1993)

References

Cribari-Neto F. (2004). "Asymptotic Inference Under Heteroskedasticity of Unknown Form." *Computational Statistics & Data Analysis*, **45**, 215-233.

Greene W.H. (1993). *Econometric Analysis*, 2nd edition. Macmillan Publishing Company, New York.

US Department of Commerce (1979). *Statistical Abstract of the United States*. US Government Printing Office, Washington, DC.

Examples

```
## Willam H. Greene, Econometric Analysis, 2nd Ed.
## Chapter 14
## load data set, p. 385, Table 14.1
data("PublicSchools", package = "sandwich")

## omit NA in Wisconsin and scale income
ps <- na.omit(PublicSchools)
ps$Income <- ps$Income * 0.0001

## fit quadratic regression, p. 385, Table 14.2
fmq <- lm(Expenditure ~ Income + I(Income^2), data = ps)
```

```
summary(fmq)

## compare standard and HC0 standard errors
## p. 391, Table 14.3
coef(fmq)
sqrt(diag(vcovHC(fmq, type = "const")))
sqrt(diag(vcovHC(fmq, type = "HC0")))


if(require(lmtest)) {
## compare t ratio
coeftest(fmq, vcov = vcovHC(fmq, type = "HC0"))

## White test, p. 393, Example 14.5
wt <- lm(residuals(fmq)^2 ~ poly(Income, 4), data = ps)
wt.stat <- summary(wt)$r.squared * nrow(ps)
c(wt.stat, pchisq(wt.stat, df = 3, lower = FALSE))

## Bresch-Pagan test, p. 395, Example 14.7
bptest(fmq, studentize = FALSE)
bptest(fmq)

## Francisco Cribari-Neto, Asymptotic Inference, CSDA 45
## quasi z-tests, p. 229, Table 8
## with Alaska
coeftest(fmq, df = Inf)[3,4]
coeftest(fmq, df = Inf, vcov = vcovHC(fmq, type = "HC0"))[3,4]
coeftest(fmq, df = Inf, vcov = vcovHC(fmq, type = "HC3"))[3,4]
coeftest(fmq, df = Inf, vcov = vcovHC(fmq, type = "HC4"))[3,4]
## without Alaska (observation 2)
fmq1 <- lm(Expenditure ~ Income + I(Income^2), data = ps[-2,])
coeftest(fmq1, df = Inf)[3,4]
coeftest(fmq1, df = Inf, vcov = vcovHC(fmq1, type = "HC0"))[3,4]
coeftest(fmq1, df = Inf, vcov = vcovHC(fmq1, type = "HC3"))[3,4]
coeftest(fmq1, df = Inf, vcov = vcovHC(fmq1, type = "HC4"))[3,4]
}

## visualization, p. 230, Figure 1
plot(Expenditure ~ Income, data = ps,
  xlab = "per capita income",
  ylab = "per capita spending on public schools")
inc <- seq(0.5, 1.2, by = 0.001)
lines(inc, predict(fmq, data.frame(Income = inc)), col = 4)
fml <- lm(Expenditure ~ Income, data = ps)
abline(fml)
text(ps[2,2], ps[2,1], rownames(ps)[2], pos = 2)
```

---

sandwich *Making Sandwiches with Bread and Meat*

Description

Constructing sandwich covariance matrix estimators by multiplying bread and meat matrices.

Usage

```
sandwich(x, bread. = bread, meat. = meat, ...)
```

Arguments

x	a fitted model object.
bread.	either a bread matrix or a function for computing this via bread.(x).
meat.	either a bread matrix or a function for computing this via meat.(x, ...).
...	arguments passed to the meat function.

Details

sandwich is a simple convenience function that takes a bread matrix (i.e., estimator of the expectation of the negative derivative of the estimating functions) and a meat matrix (i.e., estimator of the variance of the estimating functions) and multiplies them to a sandwich with meat between two slices of bread. By default bread and meat are called.

Some theoretical background along with implementation details is introduced in Zeileis (2006) and also used in Zeileis et al. (2020).

Value

A matrix containing the sandwich covariance matrix estimate. Typically, this should be an $k \times k$ matrix corresponding to k parameters.

References

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

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See Also

bread, meat, meatHC, meatHAC

Examples

```
x <- sin(1:10)
y <- rnorm(10)
fm <- lm(y ~ x)

sandwich(fm)
vcovHC(fm, type = "HC")
```

vcovBS *(Clustered) Bootstrap Covariance Matrix Estimation*

Description

Object-oriented estimation of basic bootstrap covariances, using simple (clustered) case-based resampling, plus more refined methods for `lm` and `glm` models.

Usage

```
vcovBS(x, ...)

## Default S3 method:
vcovBS(x, cluster = NULL, R = 250, start = FALSE, type = "xy", ...,
  fix = FALSE, use = "pairwise.complete.obs", applyfun = NULL, cores = NULL,
  center = "mean")

## S3 method for class 'lm'
vcovBS(x, cluster = NULL, R = 250, type = "xy", ...,
  fix = FALSE, use = "pairwise.complete.obs", applyfun = NULL, cores = NULL,
  qrjoint = FALSE, center = "mean")

## S3 method for class 'glm'
vcovBS(x, cluster = NULL, R = 250, start = FALSE, type = "xy", ...,
  fix = FALSE, use = "pairwise.complete.obs", applyfun = NULL, cores = NULL,
  center = "mean")
```

Arguments

`x`	a fitted model object.
`cluster`	a variable indicating the clustering of observations, a `list` (or `data.frame`) thereof, or a formula specifying which variables from the fitted model should be used (see examples). By default (`cluster = NULL`), either `attr(x, "cluster")` is used (if any) or otherwise every observation is assumed to be its own cluster.
`R`	integer. Number of bootstrap replications.
`start`	logical. Should `coef(x)` be passed as `start` to the `update(x, subset = ...)` call? In case the model `x` is computed by some numeric iteration, this may speed up the bootstrapping.
`type`	character (or function). The character string specifies the type of bootstrap to use: In the default and `glm` method the three types `"xy"`, `"fractional"`, and `"jackknife"` are available. In the `lm` method there are additionally `"residual"`, `"wild"` (or equivalently: `"wild-rademacher"` or `"rademacher"`), `"mammen"` (or `"wild-mammen"`), `"norm"` (or `"wild-norm"`), `"webb"` (or `"wild-webb"`). Finally, for the `lm` method `type` can be a `function(n)` for drawing wild bootstrap factors.

...	arguments passed to methods. For the default method, this is passed to update, and for the lm method to lm.fit.
fix	logical. Should the covariance matrix be fixed to be positive semi-definite in case it is not?
use	character. Specification passed to cov for handling missing coefficients/parameters.
applyfun	an optional lapply-style function with arguments function(X, FUN, ...). It is used for refitting the model to the bootstrap samples. The default is to use the basic lapply function unless the cores argument is specified (see below).
cores	numeric. If set to an integer the applyfun is set to mclapply with the desired number of cores, except on Windows where parLapply with makeCluster(cores) is used.
center	character. For type = "jackknife" the coefficients from all jacknife samples (each dropping one observational unit/cluster) can be centered by their "mean" (default) or by the original full-sample "estimate".
qrjoint	logical. For residual-based and wild boostrap (i.e., type != "xy"), should the bootstrap sample the dependent variable and then apply the QR decomposition jointly only once? If FALSE, the boostrap applies the QR decomposition separately in each iteration and samples coefficients directly. If the sample size (and the number of coefficients) is large, then qrjoint = TRUE maybe significantly faster while requiring much more memory.

Details

Clustered sandwich estimators are used to adjust inference when errors are correlated within (but not between) clusters. See the documentation for vcovCL for specifics about covariance clustering. This function allows for clustering in arbitrarily many cluster dimensions (e.g., firm, time, industry), given all dimensions have enough clusters (for more details, see Cameron et al. 2011). Unlike vcovCL, vcovBS uses a bootstrap rather than an asymptotic solution.

Basic (clustered) bootstrap covariance matrix estimation is provided by the default vcovBS method. It samples clusters (where each observation is its own cluster by default), i.e., using case-based resampling. For obtaining a covariance matrix estimate it is assumed that an update of the model with the resampled subset can be obtained, the coef extracted, and finally the covariance computed with cov.

The update model is evaluated in the environment(terms(x)) (if available). To speed up computations two further arguments can be leveraged.

1. Instead of lapply a parallelized function such as parLapply or mclapply can be specified to iterate over the bootstrap replications. For the latter, specifying cores = ... is a convenience shortcut.
2. When specifying start = TRUE, the coef(x) are passed to update as start = coef(x). This may not be supported by all model fitting functions and is hence not turned on by default.

The "xy" or "pairs" bootstrap is consistent for heteroscedasticity and clustered errors, and converges to the asymptotic solution used in vcovCL as R, n, and g become large (n and g are the number of observations and the number of clusters, respectively; see Efron 1979, or Mammen 1992, for a discussion of bootstrap asymptotics). For small g – particularly under 30 groups – the bootstrap will converge to a slightly different value than the asymptotic method, due to the limited number of

distinct bootstrap replications possible (see Webb 2014 for a discussion of this phenomonon). The bootstrap will not necessarily converge to an asymptotic estimate that has been corrected for small samples.

The xy approach to bootstrapping is generally only of interest to the practitioner when the asymptotic solution is unavailable (this can happen when using estimators that have no `estfun` function, for example). The residual bootstrap, by contrast, is rarely of practical interest, because while it provides consistent inference for clustered standard errors, it is not robust to heteroscedasticity. More generally, bootstrapping is useful when the bootstrap makes different assumptions than the asymptotic estimator, in particular when the number of clusters is small and large n or g assumptions are unreasonable. Bootstrapping is also often effective for nonlinear models, particularly in smaller samples, where asymptotic approaches often perform relatively poorly. See Cameron and Miller (2015) for further discussion of bootstrap techniques in practical applications, and Zeileis et al. (2020) show simulations comparing `vcovBS` to `vcovCL` in several settings.

The jackknife approach is of particular interest in practice because it can be shown to be exactly equivalent to the HC3 (without cluster adjustment, also known as CV3) covariance matrix estimator in linear models (see MacKinnon, Nielsen, Webb 2022). If the number of observations per cluster is large it may become impossible to compute this estimator via `vcovCL` while using the jackknife approach will still be feasible. In nonlinear models (including non-Gaussian GLMs) the jackknife and the HC3 estimator do not coincide but the jackknife might still be a useful alternative when the HC3 cannot be computed. A convenience interface `vcovJK` is provided whose default method simply calls `vcovBS(..., type = "jackknife")`.

The fractional-random-weight bootstrap (see Xu et al. 2020), first introduced by Rubin (1981) as Bayesian bootstrap, is an alternative to the xy bootstrap when it is computationally challenging or even impractical to reestimate the model on subsets, e.g., when "successes" in binary responses are rare or when the number of parameters is close to the sample size. In these situations excluding some observations completely is the source of the problems, i.e., giving some observations zero weight while others receive integer weights of one ore more. The fractional bootstrap mitigates this by giving every observation a positive fractional weight, drawn from a Dirichlet distribution. These may become close to zero but never exclude an observation completly, thus stabilizing the computation of the reweighted models.

The `glm` method works essentially like the default method but calls `glm.fit` instead of `update`.

The `lm` method provides additional bootstrapping `type`s and computes the bootstrapped coefficient estimates somewhat more efficiently using `lm.fit` (for case-based resampling) or `qr.coef` rather than `update`. The default `type` is case-based resampling (`type = "xy"`) as in the default method. Alternative `type` specifications are:

- `"residual"`. The residual cluster bootstrap resamples the residuals (as above, by cluster) which are subsequently added to the fitted values to obtain the bootstrapped response variable: $y^* = \hat{y} + e^*$. Coefficients can then be estimated using `qr.coef()`, reusing the QR decomposition from the original fit. As Cameron et al. (2008) point out, the residual cluster bootstrap is not well-defined when the clusters are unbalanced as residuals from one cluster cannot be easily assigned to another cluster with different size. Hence a warning is issued in that case.
- `"wild"` (or equivalently `"wild-rademacher"` or `"rademacher"`). The wild cluster bootstrap does not actually resample the residuals but instead reforms the dependent variable by multiplying the residual by a randomly drawn value and adding the result to the fitted value:

$y^* = \hat{y} + e \cdot w$ (see Cameron et al. 2008). By default, the factors are drawn from the Rademacher distribution: `function(n) sample(c(-1, 1), n, replace = TRUE)`.

- `"mammen"` (or `"wild-mammen"`). This draws the wild bootstrap factors as suggested by Mammen (1993): `sample(c(-1, 1) * (sqrt(5) + c(-1, 1))/2, n, replace = TRUE, prob = (sqrt(5) + c(1, -1))/(2 * sqrt(5)))`.
- `"webb"` (or `"wild-webb"`). This implements the six-point distribution suggested by Webb (2014), which may improve inference when the number of clusters is small: `sample(c(-sqrt((3:1)/2), sqrt((1:3)/2)), n, replace = TRUE)`.
- `"norm"` (or `"wild-norm"`). The standard normal/Gaussian distribution is used for drawing the wild bootstrap factors: `function(n) rnorm(n)`.
- User-defined function. This needs of the form as above, i.e., a `function(n)` returning a vector of random wild bootstrap factors of corresponding length.

Value

A matrix containing the covariance matrix estimate.

References

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See Also

vcovCL, vcovJK

Examples

```
## Petersen's data
data("PetersenCL", package = "sandwich")
m <- lm(y ~ x, data = PetersenCL)

## comparison of different standard errors
suppressWarnings(RNGversion("3.5.0"))
set.seed(1)
cbind(
  "classical" = sqrt(diag(vcov(m))),
  "HC-cluster" = sqrt(diag(vcovCL(m, cluster = ~ firm))),
  "BS-cluster" = sqrt(diag(vcovBS(m, cluster = ~ firm))),
  "FW-cluster" = sqrt(diag(vcovBS(m, cluster = ~ firm, type = "fractional")))
)

## two-way wild cluster bootstrap with Mammen distribution
vcovBS(m, cluster = ~ firm + year, type = "wild-mammen")

## jackknife estimator coincides with HC3 (aka CV3)
all.equal(
  vcovBS(m, cluster = ~ firm, type = "jackknife"),
  vcovCL(m, cluster = ~ firm, type = "HC3", cadjust = FALSE),
  tolerance = 1e-7
)
```

---

vcovCL *Clustered Covariance Matrix Estimation*

Description

Estimation of one-way and multi-way clustered covariance matrices using an object-oriented approach.

Usage

```
vcovCL(x, cluster = NULL, type = NULL, sandwich = TRUE, fix = FALSE, ...)
meatCL(x, cluster = NULL, type = NULL, cadjust = TRUE, multi0 = FALSE, ...)
```

Arguments

x	a fitted model object.
cluster	a variable indicating the clustering of observations, a list (or data.frame) thereof, or a formula specifying which variables from the fitted model should be used (see examples). By default (cluster = NULL), either attr(x, "cluster") is used (if any) or otherwise every observation is assumed to be its own cluster.

type	a character string specifying the estimation type (HC0–HC3). The default is to use "HC1" for lm objects and "HC0" otherwise.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the meat matrix is returned.
fix	logical. Should the covariance matrix be fixed to be positive semi-definite in case it is not?
cadjust	logical. Should a cluster adjustment be applied?
multi0	logical. Should the HC0 estimate be used for the final adjustment in multi-way clustered covariances?
...	arguments passed to meatCL.

Details

Clustered sandwich estimators are used to adjust inference when errors are correlated within (but not between) clusters. vcovCL allows for clustering in arbitrary many cluster dimensions (e.g., firm, time, industry), given all dimensions have enough clusters (for more details, see Cameron et al. 2011). If each observation is its own cluster, the clustered sandwich collapses to the basic sandwich covariance.

The function meatCL is the work horse for estimating the meat of clustered sandwich estimators. vcovCL is a wrapper calling sandwich and bread (Zeileis 2006). vcovCL is applicable beyond lm or glm class objects.

bread and meat matrices are multiplied to construct clustered sandwich estimators. The meat of a clustered sandwich estimator is the cross product of the clusterwise summed estimating functions. Instead of summing over all individuals, first sum over cluster.

A two-way clustered sandwich estimator M (e.g., for cluster dimensions "firm" and "industry" or "id" and "time") is a linear combination of one-way clustered sandwich estimators for both dimensions (M_{id}, M_{time}) minus the clustered sandwich estimator, with clusters formed out of the intersection of both dimensions ($M_{id \cap time}$):

$$M = M_{id} + M_{time} - M_{id \cap time}$$

. Additionally, each of the three terms can be weighted by the corresponding cluster bias adjustment factor (see below and Equation 20 in Zeileis et al. 2020). Instead of subtracting $M_{id \cap time}$ as the last subtracted matrix, Ma (2014) suggests to subtract the basic HC0 covariance matrix when only a single observation is in each intersection of id and $time$. Set multi0 = TRUE to subtract the basic HC0 covariance matrix as the last subtracted matrix in multi-way clustering. For details, see also Petersen (2009) and Thompson (2011).

With the type argument, HC0 to HC3 types of bias adjustment can be employed, following the terminology used by MacKinnon and White (1985) for heteroscedasticity corrections. HC0 applies no small sample bias adjustment. HC1 applies a degrees of freedom-based correction, $(n-1)/(n-k)$ where n is the number of observations and k is the number of explanatory or predictor variables in the model. HC1 is the most commonly used approach for linear models, and HC0 otherwise. Hence these are the defaults in vcovCL. However, HC0 and HC1 are less effective than HC2 and HC3 when the number of clusters is relatively small (Cameron et al. 2008). HC2 and HC3 types of bias adjustment are geared towards the linear model, but they are also applicable for GLMs (see Bell and McCaffrey 2002, and Kauermann and Carroll 2001, for details). A precondition for HC2

and HC3 types of bias adjustment is the availability of a hat matrix (or a weighted version therof for GLMs) and hence these two types are currently only implemented for `lm` and `glm` objects.

An alternative to the clustered HC3 estimator is the clustered jackknife estimator which is available in `vcovBS` with `type = "jackknife"`. In linear models the HC3 and the jackknife estimator coincide (MacKinnon et al. 2022) with the latter still being computationally feasible if the number of observations per cluster is large. In nonlinear models (including non-Gaussian GLMs) the jackknife and the HC3 estimator do not coincide but the jackknife might still be a useful alternative when the HC3 cannot be computed.

The `cadjust` argument allows to switch the cluster bias adjustment factor $G/(G-1)$ on and off (where G is the number of clusters in a cluster dimension g) See Cameron et al. (2008) and Cameron et al. (2011) for more details about small-sample modifications.

The `cluster` specification can be made in a number of ways: The `cluster` can be a single variable or a `list`/`data.frame` of multiple clustering variables. If `expand.model.frame` works for the model object `x`, the `cluster` can also be a `formula`. By default (`cluster = NULL`), `attr(x, "cluster")` is checked and used if available. If not, every observation is assumed to be its own cluster. If the number of observations in the model `x` is smaller than in the original `data` due to `NA` processing, then the same `NA` processing can be applied to `cluster` if necessary (and `x$na.action` being available).

Cameron et al. (2011) observe that sometimes the covariance matrix is not positive-semidefinite and recommend to employ the eigendecomposition of the estimated covariance matrix, setting any negative eigenvalue(s) to zero. This fix is applied, if necessary, when `fix = TRUE` is specified.

Value

A matrix containing the covariance matrix estimate.

References

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See Also

vcovHC, vcovBS

Examples

```
## Petersen's data
data("PetersenCL", package = "sandwich")
m <- lm(y ~ x, data = PetersenCL)

## clustered covariances
## one-way
vcovCL(m, cluster = ~ firm)
vcovCL(m, cluster = PetersenCL$firm) ## same
## one-way with HC2
vcovCL(m, cluster = ~ firm, type = "HC2")
## two-way
vcovCL(m, cluster = ~ firm + year)
vcovCL(m, cluster = PetersenCL[, c("firm", "year")]) ## same

## comparison with cross-section sandwiches
## HC0
all.equal(sandwich(m), vcovCL(m, type = "HC0", cadjust = FALSE))
## HC2
all.equal(vcovHC(m, type = "HC2"), vcovCL(m, type = "HC2"))
## HC3
all.equal(vcovHC(m, type = "HC3"), vcovCL(m, type = "HC3"))

## Innovation data
data("InstInnovation", package = "sandwich")

## replication of one-way clustered standard errors for model 3, Table I
## and model 1, Table II in Berger et al. (2017), see ?InstInnovation

## count regression formula
f1 <- cites ~ institutions + log(capital/employment) + log(sales) + industry + year

## model 3, Table I: Poisson model
```

```
## one-way clustered standard errors
tab_I_3_pois <- glm(f1, data = InstInnovation, family = poisson)
vcov_pois <- vcovCL(tab_I_3_pois, InstInnovation$company)
sqrt(diag(vcov_pois))[2:4]

## coefficient tables
if(require("lmtest")) {
coeftest(tab_I_3_pois, vcov = vcov_pois)[2:4, ]
}

## Not run:
## model 1, Table II: negative binomial hurdle model
## (requires "pscl" or alternatively "countreg" from R-Forge)
library("pscl")
library("lmtest")
tab_II_3_hurdle <- hurdle(f1, data = InstInnovation, dist = "negbin")
#  dist = "negbin", zero.dist = "negbin", separate = FALSE)
vcov_hurdle <- vcovCL(tab_II_3_hurdle, InstInnovation$company)
sqrt(diag(vcov_hurdle))[c(2:4, 149:151)]
coeftest(tab_II_3_hurdle, vcov = vcov_hurdle)[c(2:4, 149:151), ]

## End(Not run)
```

---

vcovHAC *Heteroscedasticity and Autocorrelation Consistent (HAC) Covariance Matrix Estimation*

Description

Heteroscedasticity and autocorrelation consistent (HAC) estimation of the covariance matrix of the coefficient estimates in a (generalized) linear regression model.

Usage

```
vcovHAC(x, ...)

## Default S3 method:
vcovHAC(x, order.by = NULL, prewhite = FALSE, weights = weightsAndrews,
  adjust = TRUE, diagnostics = FALSE, sandwich = TRUE, ar.method = "ols",
  data = list(), ...)

meatHAC(x, order.by = NULL, prewhite = FALSE, weights = weightsAndrews,
  adjust = TRUE, diagnostics = FALSE, ar.method = "ols", data = list(), ...)
```

Arguments

x	a fitted model object.
order.by	Either a vector z or a formula with a single explanatory variable like ~ z. The observations in the model are ordered by the size of z. If set to NULL (the default) the observations are assumed to be ordered (e.g., a time series).

prewhite	logical or integer. Should the estimating functions be prewhitened? If TRUE or greater than 0 a VAR model of order as.integer(prewhite) is fitted via ar with method "ols" and demean = FALSE.
weights	Either a vector of weights for the autocovariances or a function to compute these weights based on x, order.by, prewhite, ar.method and data. If weights is a function it has to take these arguments. See also details.
adjust	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k the number of estimated parameters.
diagnostics	logical. Should additional model diagnostics be returned? See below for details.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the meat matrix is returned.
ar.method	character. The method argument passed to ar for prewhitening.
data	an optional data frame containing the variables in the order.by model. By default the variables are taken from the environment which vcovHAC is called from.
...	arguments passed to sandwich (in vcovHAC) and estfun (in meatHAC), respectively.

Details

The function meatHAC is the real work horse for estimating the meat of HAC sandwich estimators – the default vcovHAC method is a wrapper calling sandwich and bread. See Zeileis (2006) for more implementation details. The theoretical background, exemplified for the linear regression model, is described in Zeileis (2004).

Both functions construct weighted information sandwich variance estimators for parametric models fitted to time series data. These are basically constructed from weighted sums of autocovariances of the estimating functions (as extracted by estfun). The crucial step is the specification of weights: the user can either supply vcovHAC with some vector of weights or with a function that computes these weights adaptively (based on the arguments x, order.by, prewhite and data). Two functions for adaptively choosing weights are implemented in weightsAndrews implementing the results of Andrews (1991) and in weightsLumley implementing the results of Lumley (1999). The functions kernHAC and weave respectively are to more convenient interfaces for vcovHAC with these functions.

Prewhitening based on VAR approximations is described as suggested in Andrews & Monahan (1992).

The covariance matrix estimators have been improved by the addition of a bias correction and an approximate denominator degrees of freedom for test and confidence interval construction. See Lumley & Heagerty (1999) for details.

Value

A matrix containing the covariance matrix estimate. If diagnostics was set to TRUE this has an attribute "diagnostics" which is a list with

bias.correction	multiplicative bias correction
df	Approximate denominator degrees of freedom

References

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Zeileis A (2004). "Econometric Computing with HC and HAC Covariance Matrix Estimators." *Journal of Statistical Software*, **11**(10), 1–17. doi:10.18637/jss.v011.i10

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

See Also

`weightsLumley`, `weightsAndrews`, `weave`, `kernHAC`

Examples

```
x <- sin(1:100)
y <- 1 + x + rnorm(100)
fm <- lm(y ~ x)
vcovHAC(fm)
vcov(fm)
```

---

vcovHC *Heteroscedasticity-Consistent Covariance Matrix Estimation*

Description

Heteroscedasticity-consistent estimation of the covariance matrix of the coefficient estimates in regression models.

Usage

```
vcovHC(x, ...)

## Default S3 method:
vcovHC(x,
  type = c("HC3", "const", "HC", "HC0", "HC1", "HC2", "HC4", "HC4m", "HC5"),
  omega = NULL, sandwich = TRUE, ...)

meatHC(x, type = , omega = NULL, ...)
```

Arguments

x	a fitted model object.
type	a character string specifying the estimation type. For details see below.
omega	a vector or a function depending on the arguments residuals (the working residuals of the model), diaghat (the diagonal of the corresponding hat matrix) and df (the residual degrees of freedom). For details see below.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the meat matrix is returned.
...	arguments passed to sandwich (in vcovHC) and estfun (in meatHC), respectively.

Details

The function meatHC is the real work horse for estimating the meat of HC sandwich estimators – the default vcovHC method is a wrapper calling sandwich and bread. See Zeileis (2006) for more implementation details. The theoretical background, exemplified for the linear regression model, is described below and in Zeileis (2004). Analogous formulas are employed for other types of models, provided that they depend on a single linear predictor and the estimating functions can be represented as a product of "working residual" and regressor vector (Zeileis 2006, Equation 7).

When type = "const" constant variances are assumed and and vcovHC gives the usual estimate of the covariance matrix of the coefficient estimates:

$$\hat{\sigma}^2(X^\top X)^{-1}$$

All other methods do not assume constant variances and are suitable in case of heteroscedasticity. "HC" (or equivalently "HC0") gives White's estimator, the other estimators are refinements of this. They are all of form

$$(X^\top X)^{-1} X^\top \Omega X (X^\top X)^{-1}$$

and differ in the choice of Omega. This is in all cases a diagonal matrix whose elements can be either supplied as a vector omega or as a a function omega of the residuals, the diagonal elements of the hat matrix and the residual degrees of freedom. For White's estimator

```
omega <- function(residuals, diaghat, df) residuals^2
```

Instead of specifying the diagonal omega or a function for estimating it, the type argument can be used to specify the HC0 to HC5 estimators. If omega is used, type is ignored.

Long & Ervin (2000) conduct a simulation study of HC estimators (HC0 to HC3) in the linear regression model, recommending to use HC3 which is thus the default in vcovHC. Cribari-Neto (2004), Cribari-Neto, Souza, & Vasconcellos (2007), and Cribari-Neto & Da Silva (2011), respectively, suggest the HC4, HC5, and modified HC4m type estimators. All of them are tailored to take into account the effect of leverage points in the design matrix. For more details see the references.

Value

A matrix containing the covariance matrix estimate.

References

Cribari-Neto F. (2004). "Asymptotic Inference under Heteroskedasticity of Unknown Form." *Computational Statistics & Data Analysis* **45**, 215–233.

Cribari-Neto F., Da Silva W.B. (2011). "A New Heteroskedasticity-Consistent Covariance Matrix Estimator for the Linear Regression Model." *Advances in Statistical Analysis*, **95**(2), 129–146.

Cribari-Neto F., Souza T.C., Vasconcellos, K.L.P. (2007). "Inference under Heteroskedasticity and Leveraged Data." *Communications in Statistics – Theory and Methods*, **36**, 1877–1888. Errata: **37**, 3329–3330, 2008.

Long J. S., Ervin L. H. (2000). "Using Heteroscedasticity Consistent Standard Errors in the Linear Regression Model." *The American Statistician*, **54**, 217–224.

MacKinnon J. G., White H. (1985). "Some Heteroskedasticity-Consistent Covariance Matrix Estimators with Improved Finite Sample Properties." *Journal of Econometrics*, **29**, 305–325.

White H. (1980). "A Heteroskedasticity-Consistent Covariance Matrix and a Direct Test for Heteroskedasticity." *Econometrica* **48**, 817–838.

Zeileis A (2004). "Econometric Computing with HC and HAC Covariance Matrix Estimators." *Journal of Statistical Software*, **11**(10), 1–17. doi:10.18637/jss.v011.i10

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

See Also

`lm`, `hccm`, `bptest`, `ncv.test`

Examples

```
## generate linear regression relationship
## with homoscedastic variances
x <- sin(1:100)
y <- 1 + x + rnorm(100)
## model fit and HC3 covariance
fm <- lm(y ~ x)
vcovHC(fm)
## usual covariance matrix
vcovHC(fm, type = "const")
vcov(fm)

sigma2 <- sum(residuals(lm(y ~ x))^2)/98
sigma2 * solve(crossprod(cbind(1, x)))
```

---

vcovJK *(Clustered) Jackknife Covariance Matrix Estimation*

Description

Object-oriented estimation of jackknife covariances, i.e., based on the centered outer product of leave-on-out estimates of the model coefficients/parameters.

Usage

```
vcovJK(x, ...)

## Default S3 method:
vcovJK(x, cluster = NULL, center = "mean", ...)
```

Arguments

x	a fitted model object.
cluster	a variable indicating the clustering of observations, a list (or data.frame) thereof, or a formula specifying which variables from the fitted model should be used (see examples). By default (cluster = NULL), either attr(x, "cluster") is used (if any) or otherwise every observation is assumed to be its own cluster.
center	character specifying how to center the coefficients from all jacknife samples (each dropping one observational unit/cluster). By default the coefficients are centered by their "mean" across the sample or, alternatively, by the original full-sample "estimate".
...	arguments passed to methods. For the default method, this is passed to vcovBS.

Details

Jackknife covariance estimation is based on leave-one-out estimates of the coefficients/parameters of a model. This means that the model is reestimated after dropping each observational unit once, i.e., each individual observation in independent observations or each cluster in dependent data. The covariance matrix is then constructed from the scaled outer product of the centered jackknife estimates. Centering can either be done by the mean of the jackknife coefficients (default) or by the original full-sample estimates. Scaling is done by (N - 1)/N where N is the number of observational units.

Recent research has shown that the jackknife covariance estimate have particularly useful properties in practice: they are not downward biased and yield better coverage rates for confidence intervals compared to other "robust" covariance estimates. See MacKinnon et al. (2022) and Hansen (2022) for more details.

As jackknife covariances are also based on reestimation of the coefficients on subsamples, their computation is very similar to bootstrap covariances. Hence, the vcovBS methods provided in the package all offer an argument vcovBS(..., type = "jackknife"). This is called by the default vcovJK method. Therefore, see the arguments of vcovBS for further details, e.g., for leveraging multicore computations etc.

In the linear regression model, the jackknife covariance can actually be computed without reestimating the coefficients but using only the full-sample estimates and certain elements of the so-called hat matrix. Namly the diagonal elements or blocks of elements from the hat matrix are needed for independent observations and clustered data, respectively. These alternative computations of the jackknife covariances are available in vcovHC and vcovCL, respectively, in both cases with argument type = "HC3". To obtain HC3 covariances that exactly match the jackknife covariances, the jackknife has to be centered with the full-sample estimates and the right finite-sample adjustment has to be selected for the HC3.

In small to moderate sample sizes, the HC3 estimation techniques are typically much faster than the jackknife. However, in large samples it may become impossible to compute the HC3 covariances while the jackknife approach is still feasible.

Value

A matrix containing the covariance matrix estimate.

References

Bell RM, McCaffrey DF (2002). "Bias Reduction in Standard Errors for Linear Regression with Multi-Stage Samples", *Survey Methodology*, **28**(2), 169–181.

Hansen BE (2022). "Jackknife Standard Errors for Clustered Regression", Working Paper, August 2022. https://www.ssc.wisc.edu/~bhansen/papers/tcauchy.html

MacKinnon JG, Nielsen MØ, Webb MD (2022). "Cluster-Robust Inference: A Guide to Empirical Practice", *Journal of Econometrics*, Forthcoming. doi:10.1016/j.jeconom.2022.04.001

Zeileis A, Köll S, Graham N (2020). "Various Versatile Variances: An Object-Oriented Implementation of Clustered Covariances in R." *Journal of Statistical Software*, **95**(1), 1–36. doi:10.18637/jss.v095.i01

See Also

vcovJK, vcovHC, vcovCL

Examples

```
## cross-section data
data("PublicSchools", package = "sandwich")
m1 <- lm(Expenditure ~ poly(Income, 2), data = PublicSchools)
vcovJK(m1, center = "estimate")
vcovHC(m1, type = "HC3") * (nobs(m1) - 1)/nobs(m1)

## clustered data
data("PetersenCL", package = "sandwich")
m2 <- lm(y ~ x, data = PetersenCL)

## jackknife estimator coincides with HC3 (aka CV3)
vcovJK(m2, cluster = ~ firm, center = "estimate")
vcovCL(m2, cluster = ~ firm, type = "HC3", cadjust = FALSE)
```

---

vcovOPG *Outer-Product-of-Gradients Covariance Matrix Estimation*

Description

Outer product of gradients estimation for the covariance matrix of the coefficient estimates in regression models.

Usage

```
vcovOPG(x, adjust = FALSE, ...)
```

Arguments

x	a fitted model object.
adjust	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k the number of estimated parameters.
...	arguments passed to the estfun function.

Details

In correctly specified models, the "meat" matrix (cross product of estimating functions, see meat) and the inverse of the "bread" matrix (inverse of the derivative of the estimating functions, see bread) are equal and correspond to the Fisher information matrix. Typically, an empirical version of the bread is used for estimation of the information but alternatively it is also possible to use the meat. This method is also known as the outer product of gradients (OPG) estimator (Cameron & Trivedi 2005).

Using the **sandwich** infrastructure, the OPG estimator could easily be computed via `solve(meat(obj))` (modulo scaling). To employ numerically more stable implementation of the inversion, this simple convenience function can be used: `vcovOPG(obj)`.

Note that this only works if the `estfun()` method computes the maximum likelihood scores (and not a scaled version such as least squares scores for `"lm"` objects).

Value

A matrix containing the covariance matrix estimate.

References

Cameron AC and Trivedi PK (2005). *Microeconometrics: Methods and Applications*. Cambridge University Press, Cambridge.

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators." *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

See Also

meat, bread, sandwich

Examples

```
## generate poisson regression relationship
x <- sin(1:100)
y <- rpois(100, exp(1 + x))
## compute usual covariance matrix of coefficient estimates
fm <- glm(y ~ x, family = poisson)
vcov(fm)
vcovOPG(fm)
```

vcovPC *Panel-Corrected Covariance Matrix Estimation*

Description

Estimation of sandwich covariances a la Beck and Katz (1995) for panel data.

Usage

```
vcovPC(x, cluster = NULL, order.by = NULL,
  pairwise = FALSE, sandwich = TRUE, fix = FALSE, ...)

meatPC(x, cluster = NULL, order.by = NULL,
  pairwise = FALSE, kronecker = TRUE, ...)
```

Arguments

x	a fitted model object.
cluster	a single variable indicating the clustering of observations, or a list (or data.frame) of one or two variables, or a formula specifying which one ore two variables from the fitted model should be used (see examples). In case two variables are specified, the second variable is assumed to provide the time ordering (instead of using the argument order.by). By default (cluster = NULL), either attr(x, "cluster") is used (if any) or otherwise every observation is assumed to be its own cluster.
order.by	a variable, list/data.frame, or formula indicating the aggregation within time periods. By default attr(x, "order.by") is used (if any) or specified through the second variable in cluster (see above). If neither is available, observations within clusters are assumed to be ordered.
pairwise	logical. For unbalanced panels. Indicating whether the meat should be estimated pair- or casewise.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the meat matrix is returned.
fix	logical. Should the covariance matrix be fixed to be positive semi-definite in case it is not?
kronecker	logical. Calculate the meat via the Kronecker-product, shortening the computation time for small matrices. For large matrices, set kronecker = FALSE.
...	arguments passed to the meatPC or estfun function, respectively.

Details

vcovPC is a function for estimating Beck and Katz (1995) panel-corrected covariance matrix.

The function meatPC is the work horse for estimating the meat of Beck and Katz (1995) covariance matrix estimators. vcovPC is a wrapper calling sandwich and bread (Zeileis 2006).

Following Bailey and Katz (2011), there are two alternatives to estimate the meat for unbalanced panels. For `pairwise = FALSE`, a balanced subset of the panel is used, whereas for `pairwise = TRUE`, a pairwise balanced sample is employed.

The `cluster`/`order.by` specification can be made in a number of ways: Either both can be a single variable or `cluster` can be a `list`/`data.frame` of two variables. If `expand.model.frame` works for the model object `x`, the `cluster` (and potentially additionally `order.by`) can also be a `formula`. By default (`cluster = NULL, order.by = NULL`), `attr(x, "cluster")` and `attr(x, "order.by")` are checked and used if available. If not, every observation is assumed to be its own cluster, and observations within clusters are assumed to be ordered accordingly. If the number of observations in the model `x` is smaller than in the original `data` due to `NA` processing, then the same `NA` processing can be applied to `cluster` if necessary (and `x$na.action` being available).

Value

A matrix containing the covariance matrix estimate.

References

Bailey D, Katz JN (2011). "Implementing Panel-Corrected Standard Errors in R: The pcse Package", *Journal of Statistical Software, Code Snippets*, **42**(1), 1–11. doi:10.18637/jss.v042.c01

Beck N, Katz JN (1995). "What To Do (and Not To Do) with Time-Series-Cross-Section Data in Comparative Politics", *American Political Science Review*, **89**(3), 634–647. doi:10.2307/2082979

Zeileis A (2004). "Econometric Computing with HC and HAC Covariance Matrix Estimator", *Journal of Statistical Software*, **11**(10), 1–17. doi:10.18637/jss.v011.i10

Zeileis A (2006). "Object-Oriented Computation of Sandwich Estimators", *Journal of Statistical Software*, **16**(9), 1–16. doi:10.18637/jss.v016.i09

Zeileis A, Köll S, Graham N (2020). "Various Versatile Variances: An Object-Oriented Implementation of Clustered Covariances in R." *Journal of Statistical Software*, **95**(1), 1–36. doi:10.18637/jss.v095.i01

See Also

`vcovCL`, `vcovPL`

Examples

```
## Petersen's data
data("PetersenCL", package = "sandwich")
m <- lm(y ~ x, data = PetersenCL)

## Beck and Katz (1995) standard errors
## balanced panel
sqrt(diag(vcovPC(m, cluster = ~ firm + year)))

## unbalanced panel
PU <- subset(PetersenCL, !(firm == 1 & year == 10))
pu_lm <- lm(y ~ x, data = PU)
sqrt(diag(vcovPC(pu_lm, cluster = ~ firm + year, pairwise = TRUE)))
sqrt(diag(vcovPC(pu_lm, cluster = ~ firm + year, pairwise = FALSE)))
```

```
## the following specifications of cluster/order.by are equivalent
vcovPC(m, cluster = ~ firm + year)
vcovPC(m, cluster = PetersenCL[, c("firm", "year")])
vcovPC(m, cluster = ~ firm, order.by = ~ year)
vcovPC(m, cluster = PetersenCL$firm, order.by = PetersenCL$year)

## these are also the same when observations within each
## cluster are already ordered
vcovPC(m, cluster = ~ firm)
vcovPC(m, cluster = PetersenCL$firm)
```

---

vcovPL *Clustered Covariance Matrix Estimation for Panel Data*

Description

Estimation of sandwich covariances a la Newey-West (1987) and Driscoll and Kraay (1998) for panel data.

Usage

```
vcovPL(x, cluster = NULL, order.by = NULL,
  kernel = "Bartlett", sandwich = TRUE, fix = FALSE, ...)

meatPL(x, cluster = NULL, order.by = NULL,
  kernel = "Bartlett", lag = "NW1987", bw = NULL,
  adjust = TRUE, aggregate = TRUE, ...)
```

Arguments

x	a fitted model object.
cluster	a single variable indicating the clustering of observations, or a list (or data.frame) of one or two variables, or a formula specifying which one ore two variables from the fitted model should be used (see examples). In case two variables are specified, the second variable is assumed to provide the time ordering (instead of using the argument order.by). By default (cluster = NULL), either attr(x, "cluster") is used (if any) or otherwise every observation is assumed to be its own cluster.
order.by	a variable, list/data.frame, or formula indicating the aggregation within time periods. By default attr(x, "order.by") is used (if any) or specified through the second variable in cluster (see above). If neither is available, observations within clusters are assumed to be ordered.
kernel	a character specifying the kernel used. All kernels described in Andrews (1991) are supported, see kweights.

lag	character or numeric, indicating the lag length used. Three rules of thumb ("max" or equivalently "P2009", "NW1987", or "NW1994") can be specified, or a numeric number of lags can be specified directly. By default, "NW1987" is used.
bw	numeric. The bandwidth of the kernel which by default corresponds to lag + 1. Only one of lag and bw should be used.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the meat matrix is returned.
fix	logical. Should the covariance matrix be fixed to be positive semi-definite in case it is not?
adjust	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k is the number of estimated parameters.
aggregate	logical. Should the estfun be aggregated within each time period (yielding Driscoll and Kraay 1998) or not (restricting cross-sectional and cross-serial correlation to zero, yielding panel Newey-West)?
...	arguments passed to the metaPL or estfun function, respectively.

Details

vcovPL is a function for estimating the Newey-West (1987) and Driscoll and Kraay (1998) covariance matrix. Driscoll and Kraay (1998) apply a Newey-West type correction to the sequence of cross-sectional averages of the moment conditions (see Hoechle (2007)). For large T (and regardless of the length of the cross-sectional dimension), the Driscoll and Kraay (1998) standard errors are robust to general forms of cross-sectional and serial correlation (Hoechle (2007)). The Newey-West (1987) covariance matrix restricts the Driscoll and Kraay (1998) covariance matrix to no cross-sectional correlation.

The function meatPL is the work horse for estimating the meat of Newey-West (1987) and Driscoll and Kraay (1998) covariance matrix estimators. vcovPL is a wrapper calling sandwich and bread (Zeileis 2006).

Default lag length is the "NW1987". For lag = "NW1987", the lag length is chosen from the heuristic $floor[T^{(1/4)}]$. More details on lag length selection in Hoechle (2007). For lag = "NW1994", the lag length is taken from the first step of Newey and West's (1994) plug-in procedure.

The cluster/order.by specification can be made in a number of ways: Either both can be a single variable or cluster can be a list/data.frame of two variables. If expand.model.frame works for the model object x, the cluster (and potentially additionally order.by) can also be a formula. By default (cluster = NULL, order.by = NULL), attr(x, "cluster") and attr(x, "order.by") are checked and used if available. If not, every observation is assumed to be its own cluster, and observations within clusters are assumed to be ordered accordingly. If the number of observations in the model x is smaller than in the original data due to NA processing, then the same NA processing can be applied to cluster if necessary (and x$na.action being available).

Value

A matrix containing the covariance matrix estimate.

References

Andrews DWK (1991). "Heteroscedasticity and Autocorrelation Consistent Covariance Matrix Estimation", *Econometrica*, 817–858.

Driscoll JC & Kraay AC (1998). "Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data", *The Review of Economics and Statistics*, **80**(4), 549–560.

Hoechle D (2007). "Robust Standard Errors for Panel Regressions with Cross-Sectional Dependence", *Stata Journal*, **7**(3), 281–312.

Newey WK & West KD (1987). "Hypothesis Testing with Efficient Method of Moments Estimation", *International Economic Review*, 777-787.

Newey WK & West KD (1994). "Automatic Lag Selection in Covariance Matrix Estimation", *The Review of Economic Studies*, **61**(4), 631–653.

White H (1980). "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity", *Econometrica*, 817–838. doi:10.2307/1912934

Zeileis A (2004). "Econometric Computing with HC and HAC Covariance Matrix Estimator", *Journal of Statistical Software*, **11**(10), 1–17. doi:10.18637/jss.v011.i10

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Zeileis A, Köll S, Graham N (2020). "Various Versatile Variances: An Object-Oriented Implementation of Clustered Covariances in R." *Journal of Statistical Software*, **95**(1), 1–36. doi:10.18637/jss.v095.i01

See Also

vcovCL, vcovPC

Examples

```
## Petersen's data
data("PetersenCL", package = "sandwich")
m <- lm(y ~ x, data = PetersenCL)

## Driscoll and Kraay standard errors
## lag length set to: T - 1 (maximum lag length)
## as proposed by Petersen (2009)
sqrt(diag(vcovPL(m, cluster = ~ firm + year, lag = "max", adjust = FALSE)))

## lag length set to: floor(4 * (T / 100)^(2/9))
## rule of thumb proposed by Hoechle (2007) based on Newey & West (1994)
sqrt(diag(vcovPL(m, cluster = ~ firm + year, lag = "NW1994")))

## lag length set to: floor(T^(1/4))
## rule of thumb based on Newey & West (1987)
sqrt(diag(vcovPL(m, cluster = ~ firm + year, lag = "NW1987")))

## the following specifications of cluster/order.by are equivalent
vcovPL(m, cluster = ~ firm + year)
vcovPL(m, cluster = PetersenCL[, c("firm", "year")])
vcovPL(m, cluster = ~ firm, order.by = ~ year)
```

```
vcovPL(m, cluster = PetersenCL$firm, order.by = PetersenCL$year)

## these are also the same when observations within each
## cluster are already ordered
vcovPL(m, cluster = ~ firm)
vcovPL(m, cluster = PetersenCL$firm)
```

---

weightsAndrews *Kernel-based HAC Covariance Matrix Estimation*

Description

A set of functions implementing a class of kernel-based heteroscedasticity and autocorrelation consistent (HAC) covariance matrix estimators as introduced by Andrews (1991).

Usage

```
kernHAC(x, order.by = NULL, prewhite = 1, bw = bwAndrews,
 kernel = c("Quadratic Spectral", "Truncated", "Bartlett", "Parzen", "Tukey-Hanning"),
  approx = c("AR(1)", "ARMA(1,1)"), adjust = TRUE, diagnostics = FALSE,
 sandwich = TRUE, ar.method = "ols", tol = 1e-7, data = list(), verbose = FALSE, ...)

weightsAndrews(x, order.by = NULL, bw = bwAndrews,
 kernel = c("Quadratic Spectral", "Truncated", "Bartlett", "Parzen", "Tukey-Hanning"),
 prewhite = 1, ar.method = "ols", tol = 1e-7, data = list(), verbose = FALSE, ...)

bwAndrews(x, order.by = NULL, kernel = c("Quadratic Spectral", "Truncated",
  "Bartlett", "Parzen", "Tukey-Hanning"), approx = c("AR(1)", "ARMA(1,1)"),
  weights = NULL, prewhite = 1, ar.method = "ols", data = list(), ...)
```

Arguments

x	a fitted model object. For bwAndrews it can also be a score matrix (as returned by estfun) directly.
order.by	Either a vector z or a formula with a single explanatory variable like ~ z. The observations in the model are ordered by the size of z. If set to NULL (the default) the observations are assumed to be ordered (e.g., a time series).
prewhite	logical or integer. Should the estimating functions be prewhitened? If TRUE or greater than 0 a VAR model of order as.integer(prewhite) is fitted via ar with method "ols" and demean = FALSE. The default is to use VAR(1) prewhitening.
bw	numeric or a function. The bandwidth of the kernel (corresponds to the truncation lag). If set to to a function (the default is bwAndrews) it is adaptively chosen.
kernel	a character specifying the kernel used. All kernels used are described in Andrews (1991).

approx	a character specifying the approximation method if the bandwidth bw has to be chosen by bwAndrews.
adjust	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k the number of estimated parameters.
diagnostics	logical. Should additional model diagnostics be returned? See vcovHAC for details.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the middle matrix is returned.
ar.method	character. The method argument passed to ar for prewhitening (only, not for bandwidth selection).
tol	numeric. Weights that exceed tol are used for computing the covariance matrix, all other weights are treated as 0.
data	an optional data frame containing the variables in the order.by model. By default the variables are taken from the environment which the function is called from.
verbose	logical. Should the bandwidth parameter used be printed?
...	further arguments passed to bwAndrews.
weights	numeric. A vector of weights used for weighting the estimated coefficients of the approximation model (as specified by approx). By default all weights are 1 except that for the intercept term (if there is more than one variable).

Details

kernHAC is a convenience interface to vcovHAC using weightsAndrews: first a weights function is defined and then vcovHAC is called.

The kernel weights underlying weightsAndrews are directly accessible via the function kweights and require the specification of the bandwidth parameter bw. If this is not specified it can be chosen adaptively by the function bwAndrews (except for the "Truncated" kernel). The automatic bandwidth selection is based on an approximation of the estimating functions by either AR(1) or ARMA(1,1) processes. To aggregate the estimated parameters from these approximations a weighted sum is used. The weights in this aggregation are by default all equal to 1 except that corresponding to the intercept term which is set to 0 (unless there is no other variable in the model) making the covariance matrix scale invariant.

Further details can be found in Andrews (1991).

The estimator of Newey & West (1987) is a special case of the class of estimators introduced by Andrews (1991). It can be obtained using the "Bartlett" kernel and setting bw to lag + 1. A convenience interface is provided in NeweyWest.

Value

kernHAC returns the same type of object as vcovHAC which is typically just the covariance matrix.

weightsAndrews returns a vector of weights.

bwAndrews returns the selected bandwidth parameter.

References

Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica*, **59**, 817–858.

Newey WK & West KD (1987). "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*, **55**, 703–708.

See Also

vcovHAC, NeweyWest, weightsLumley, weave

Examples

```
curve(kweights(x, kernel = "Quadratic", normalize = TRUE),
      from = 0, to = 3.2, xlab = "x", ylab = "k(x)")
curve(kweights(x, kernel = "Bartlett", normalize = TRUE),
      from = 0, to = 3.2, col = 2, add = TRUE)
curve(kweights(x, kernel = "Parzen", normalize = TRUE),
      from = 0, to = 3.2, col = 3, add = TRUE)
curve(kweights(x, kernel = "Tukey", normalize = TRUE),
      from = 0, to = 3.2, col = 4, add = TRUE)
curve(kweights(x, kernel = "Truncated", normalize = TRUE),
      from = 0, to = 3.2, col = 5, add = TRUE)

## fit investment equation
data(Investment)
fm <- lm(RealInv ~ RealGNP + RealInt, data = Investment)

## compute quadratic spectral kernel HAC estimator
kernHAC(fm)
kernHAC(fm, verbose = TRUE)

## use Parzen kernel instead, VAR(2) prewhitening, no finite sample
## adjustment and Newey & West (1994) bandwidth selection
kernHAC(fm, kernel = "Parzen", prewhite = 2, adjust = FALSE,
  bw = bwNeweyWest, verbose = TRUE)

## compare with estimate under assumption of spheric errors
vcov(fm)
```

---

weightsLumley *Weighted Empirical Adaptive Variance Estimation*

Description

A set of functions implementing weighted empirical adaptive variance estimation (WEAVE) as introduced by Lumley and Heagerty (1999). This is implemented as a special case of the general class of kernel-based heteroscedasticity and autocorrelation consistent (HAC) covariance matrix estimators as introduced by Andrews (1991), using a special choice of weights.

Usage

```
weave(x, order.by = NULL, prewhite = FALSE, C = NULL,
  method = c("truncate", "smooth"), acf = isoacf, adjust = FALSE,
  diagnostics = FALSE, sandwich = TRUE, tol = 1e-7, data = list(), ...)

weightsLumley(x, order.by = NULL, C = NULL,
  method = c("truncate", "smooth"), acf = isoacf, tol = 1e-7, data = list(), ...)
```

Arguments

x	a fitted model object.
order.by	Either a vector z or a formula with a single explanatory variable like ~ z. The observations in the model are ordered by the size of z. If set to NULL (the default) the observations are assumed to be ordered (e.g., a time series).
prewhite	logical or integer. Should the estimating functions be prewhitened? If TRUE or greater than 0 a VAR model of order as.integer(prewhite) is fitted via ar with method "ols" and demean = FALSE.
C	numeric. The cutoff constant C is by default 4 for method "truncate" and 1 for method "smooth".
method	a character specifying the method used, see details.
acf	a function that computes the autocorrelation function of a vector, by default isoacf is used.
adjust	logical. Should a finite sample adjustment be made? This amounts to multiplication with $n/(n-k)$ where n is the number of observations and k the number of estimated parameters.
diagnostics	logical. Should additional model diagnostics be returned? See vcovHAC for details.
sandwich	logical. Should the sandwich estimator be computed? If set to FALSE only the middle matrix is returned.
tol	numeric. Weights that exceed tol are used for computing the covariance matrix, all other weights are treated as 0.
data	an optional data frame containing the variables in the order.by model. By default the variables are taken from the environment which the function is called from.
...	currently not used.

Details

weave is a convenience interface to vcovHAC using weightsLumley: first a weights function is defined and then vcovHAC is called.

Both weighting methods are based on some estimate of the autocorrelation function ρ (as computed by acf) of the residuals of the model x. The weights for the "truncate" method are

$$I\{n\rho^2 > C\}$$

and the weights for the "smooth" method are

$$\min\{1, Cn\rho^2\}$$

where n is the number of observations in the model an C is the truncation constant C.

Further details can be found in Lumley & Heagerty (1999).

Value

weave returns the same type of object as vcovHAC which is typically just the covariance matrix.

weightsLumley returns a vector of weights.

References

Lumley T & Heagerty P (1999). "Weighted Empirical Adaptive Variance Estimators for Correlated Data Regression." *Journal of the Royal Statistical Society B*, **61**, 459–477.

See Also

vcovHAC, weightsAndrews, kernHAC

Examples

```
x <- sin(1:100)
y <- 1 + x + rnorm(100)
fm <- lm(y ~ x)
weave(fm)
vcov(fm)
```

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