Package 'GWnorm'

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Description Computes G-Wishart normalising constants through a Fourier approach. Either exact analytical results, numerical integration or Monte Carlo estimation are employed. Details at C. Wong, G. Moffa and J. Kuipers (2024), <doi:10.48550/arXiv.2404.06803>. Also includes approximations of the ratio of normalising constants, see details at C. Wong, G. Moffa and J. Kuipers (2025), <doi:10.48550/arXiv.2503.13046>.

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`check_prime_connected` *Prime and connected*

Description

This function just checks if a graph is connected and prime

Usage

```
check_prime_connected(graph, check_prime = TRUE)
```

Arguments

`graph`	An igraph object
`check_prime`	Boolean flag to check primality too (default TRUE)

Value

A boolean, TRUE if connected and prime

`C_GtoI_G` *G-Wishart normalising constant reparametrisation the other way*

Description

This function returns the log of the G-Wishart normalising constant I_G(beta, D) = int_[S^p_++(G)] det(K)^beta * exp(-tr(KD)) dK from the transformed version log(C_G(delta, D)) = int_[S^p_++(G)] det(K)^((delta-2)/2) * exp(-tr(KD)/2) dK

Usage

```
C_GtoI_G(graph, CGval, delta)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
CGval	A number log(C_G(delta, D))
delta	A constant > 0

Value

A number log(I_G(beta, D))

Iss_cmat *Isserlis complement matrix*

Description

This function returns the Isserlis matrix (rows/columns arranged in some order) with respect to a matrix and a graph for the missing edges (complement of the graph).

Usage

```
Iss_cmat(M, graph)
```

Arguments

M	An p by p matrix
graph	An igraph object, corresponding to the graph

Value

An m by m matrix, where m is the number of missing edges in the graph

Examples

```
# example code
adj <- matrix(0, nrow = 5, ncol = 5)
adj[1,5] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
Iss_cmat(matrix(1:25, nrow = 5, byrow = TRUE), graph)
```

Iss_mat *Isserlis matrix*

Description

This function returns the Isserlis matrix (rows/columns arranged in some order) with respect to a matrix and a graph

Usage

```
Iss_mat(M, graph)
```

Arguments

M	An p by p matrix
graph	An igraph object, corresponding to the graph

Value

An m by m matrix, where m is the number of vertices and edges in the graph

Examples

```
# example code
adj <- matrix(0, nrow = 5, ncol = 5)
adj[1,5] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
Iss_mat(matrix(1:25, nrow = 5, byrow = TRUE), graph)
```

is_6_cycle *Determine whether the graph is the cycle of length 6 or its complement*

Description

Input an igraph object, this function tells you whether graph is the cycle of length 6, or its complement.

Usage

```
is_6_cycle(graph)
```

Arguments

graph	An igraph object

Value

An integer, 1 means the graph is the cycle of length 6, 2 means the the graph is the complement of the cycle of length 6, 0 means otherwise

Examples

```
# C6 example 5 1 3 2 4 6
is_6_cycle(igraph::make_graph(edges = c(1,3, 1,5, 2,3, 2,4, 5,6, 6,4),
  n = 6, directed = FALSE))
# C6 complement example
is_6_cycle(igraph::make_graph(edges = c(1,2, 1,4, 1,6, 2,5, 2,6, 3,4,
  3,5, 3,6, 4,5), n = 6, directed = FALSE))
# not C6 examples
is_6_cycle(igraph::make_graph(edges = c(1,3, 1,5, 2,3, 2,4, 5,6, 6,4),
  n = 7, directed = FALSE))
is_6_cycle(igraph::make_graph(edges = c(1,4, 1,5, 2,3, 2,4, 5,6, 6,4),
  n = 6, directed = FALSE))
```

---

is_k_partite *Determine whether the graph is complete k partite*

Description

Input an igraph object, this function tells you the size of the partition if the graph is a k partite graph.

Usage

```
is_k_partite(graph)
```

Arguments

graph	An igraph object

Value

A vector of the size of the partition. The length of the vector is k. Empty vector means that the graph is not a complete k partite graph.

Examples

```
# not complete k-partite example
is_k_partite(igraph::make_graph(edges = c(1,2, 2,4, 2,5, 1,3, 3,5, 5,6, 6,7,
  5,7, 4,7), n = 7, directed = FALSE))
# complete 4-partite example, (1,5), (2,4), (3,7), (6)
is_k_partite(igraph::make_graph(edges = c(1,2, 1,3, 1,4, 1,6, 1,7, 2,3, 2,5, 2,6,
  2,7, 3,4, 3,5, 3,6, 4,5, 4,6, 4,7, 5,6, 5,7, 6,7), n = 7, directed = FALSE))
```

I_Gnorm *G Wishart normalising constant*

Description

This function returns the log of transformed G-Wishart normalising constant I_G(beta, D) = int_[S^p_++(G)] det(K)^beta * exp(-tr(KD)) dK for graphs G. Graphs are decomposed into connected prime components. For each component if there is no explicit formula, or the integral cannot be reduced into one dimension, MC integration is used.

Usage

```
I_Gnorm(
  graph,
  beta,
  D,
  connected = FALSE,
  prime = FALSE,
  chordal = NULL,
  n_samp = 1000,
  nu = 10,
  robust = FALSE,
  quench_k = 0,
  err_flag = FALSE,
  useBDgraph = FALSE
)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix
connected	Whether the graph is connected, if known (default FALSE)
prime	Whether the graph is prime, if known (default FALSE)
chordal	Optional. An igraph object, corresponding to a chordal completion of graph
n_samp	number of sample points used in MC integration
nu	degree of freedom of the MC importance distribution (nu=0 is Gaussian)
robust	flag of whether to use robust means (default not to, but may offer numerical stability at the risk of bias)
quench_k	scaling factor of von Mises quenching (default value of 0 means not used, but value around 1 may offer numerical stability at the risk of bias)
err_flag	flag of whether to return the log relative error estimate (or not, default), only considered for prime connected graphs
useBDgraph	flag of whether to use BDgraph instead (or not, default), useful wrapper for comparisons

Value

A number log(I_G(beta, D))

Examples

```
set.seed(42)
p <- 11
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
beta <- 1
graph <- igraph::make_graph(edges = c(1,2, 2,4, 2,5, 1,3, 3,5, # disconnected
                                      5,6, 6,7, 5,7, 4,7, # with prime decomposition too
                                      8,9, 8,11, 9,10, 10,11), n = 11, directed = FALSE)
I_Gnorm(graph, beta, D)
# compare with BDgraph
I_Gnorm(graph, beta, D, n_samp = 1e5, useBDgraph = TRUE)
```

I_GtoC_G *G-Wishart normalising constant reparametrisation one way*

Description

This function returns the log of the G-Wishart normalising constant log(C_G(delta, D)) = int_[S^p_++(G)] det(K)^((delta-2)/2) * exp(-tr(KD)/2) dK from the transformed version I_G(beta, D) = int_[S^p_++(G)] det(K)^beta * exp(-tr(KD)) dK

Usage

```
I_GtoC_G(graph, IGval, beta)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
IGval	A number log(I_G(beta, D))
beta	A constant > 0

Value

A number log(C_G(delta, D))

I_G_BD *G Wishart normalising constant with BDgraph*

Description

This function is a wrapper for BDgraph to compare and to avoid the NOTE in the package checks since we only had BDgraph in the examples

Usage

```
I_G_BD(graph, beta, D, n_samp = 1000, C_G_flag = FALSE)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix
n_samp	number of sample points used in MC integration
C_G_flag	boolean whether to return C_G instead of I_G

Value

A number log(I_G(beta, D)) [or log(C_G(delta, D))]

I_G_chordal *G Wishart normalising constant for chordal graphs*

Description

This function returns the log of transformed G-Wishart normalising constant I_G(beta, D) = int_[S^p_++(G)] det(K)^beta * exp(-tr(KD)) dK for chordal graphs G, using explicit formula (cliques / separators).

Usage

```
I_G_chordal(
  graph,
  beta,
  D = NULL,
  cliques = NULL,
  separators = NULL,
  ratio = FALSE
)
```

Arguments

graph	An igraph object, corresponding to a chordal graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix (default, NULL for identity matrix)
cliques	A list of cliques (if known)
separators	A list of separators (if known)
ratio	Whether to output just the ratio compared to the value with I (default FALSE)

Value

A number log(I_G(beta, D))

Examples

```
set.seed(42)
p <- 5
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
beta <- 1
adj <- matrix(0, nrow = p, ncol = p)
adj[1,c(2:5)] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # chordal completion of 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_chordal(graph, beta, D)
# compare with BDgraph gnorm()
I_G_BD(graph, beta, D, n_samp = 1e4)
```

I_G_complete *G Wishart normalising constant for complete graphs*

Description

This function returns the log of the transformed G-Wishart normalising constant I_G(beta, D) = int_[S^p_++(G)] det(K)^beta * exp(-tr(KD)) dK for complete graphs G, using explicit formula det(D)^(-beta-(p+1)/2) Gamma_n(beta+(p+1)/2)

Usage

```
I_G_complete(p, beta, D = NULL, ratio = FALSE)
```

Arguments

p	A positive integer, indicating the number of vertices of the graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix (default, NULL, if for identity matrix)
ratio	Whether to output just the ratio compared to the value with I (default FALSE)

Value

A number log(I_G(beta, D))

Examples

```
set.seed(42)
p <- 6
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
beta <- 1
I_G_complete(6, beta, D)
# compare with BDgraph gnorm()
adj <- matrix(1, nrow = 6, ncol = 6)
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_BD(graph, beta, D)
```

I_G_MC *G Wishart normalising constant through MC integration*

Description

This function returns the log of transformed G-Wishart normalising constant I_G(beta, D) = int_[S^p_++(G)] det(K)^beta * exp(-tr(KD)) dK for connected graphs G. If there is no explicit formula, or the integral cannot be reduced into lower dimensions, MC integration is used.

Usage

```
I_G_MC(
  graph,
  beta,
  D,
  n_samp = 1000,
  nu = 10,
  err_flag = FALSE,
  int1d = TRUE,
  robust = FALSE,
  quench_k = 0,
  chordal = NULL
)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix
n_samp	number of sample points used in MC integration

nu	degree of freedom of the MC importance distribution (nu=0 is Gaussian)
err_flag	flag of whether to return the log relative error estimate (or not, default)
int1d	flag of whether to integrate numerically in 1d (default, MC integration otherwise)
robust	flag of whether to use robust means (default not to, but may offer numerical stability at the risk of bias)
quench_k	scaling factor of von Mises quenching (default not used, value of 0, but value around 1 may offer numerical stability at the risk of bias)
chordal	Optional. An igraph object, corresponding to a chordal completion of graph

Value

A number log(I_G(beta, D))

Examples

```
set.seed(42)
p <- 5
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
beta <- 1
adj <- matrix(0, nrow = p, ncol = p)
adj[1,5] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_MC(graph, beta, D)
# compare with BDgraph gnorm()
I_G_BD(graph, beta, D, n_samp = 1e5)
```

I_G_ratio_approx *G Wishart normalising constant ratio approximation*

Description

This function returns the approximation of the ratio of log transformed G-Wishart normalising constants I_G(beta, D) / I_G(beta, I) for graphs G. Graphs are decomposed into connected prime components. For each component if there is no explicit formula, the approximation is used. Note that this is the same as the ratio C_G(delta, D) / C_G(delta, I) with delta = 2*beta + 2

Usage

```
I_G_ratio_approx(
  graph,
  beta,
  D,
  use_nlo = FALSE,
  connected = FALSE,
  prime = FALSE,
```

```
  chordal = NULL,
  use_comp = NULL
)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix
use_nlo	Whether to use the next-order approximation (TRUE) or not (default, FALSE). It is more accurate, but slower, and overwrites use_comp (as TRUE)
connected	Whether the graph is connected, if known (default FALSE)
prime	Whether the graph is prime, if known (default FALSE)
chordal	Optional. An igraph object, corresponding to a chordal completion of graph
use_comp	Whether to approximate using the graph (FALSE), the complement (TRUE) or the more efficient (default NULL)

Value

A number approximating log(I_G(beta, D)) - log(I_G(beta, I))

Examples

```
set.seed(42)
p <- 11
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
beta <- 1
graph <- igraph::make_graph(edges = c(1,2, 2,4, 2,5, 1,3, 3,5, # disconnected
                                      5,6, 6,7, 5,7, 4,7, # with prime decomposition too
                                      8,9, 8,11, 9,10, 10,11), n = 11, directed = FALSE)
I_G_ratio_approx(graph, beta, D)
I_G_ratio_approx(graph, beta, D, use_nlo = TRUE)
# compare with MC estimates
I_G_MC(graph, beta, D, n_samp = 1e5) - I_G_MC(graph, beta, diag(p), n_samp = 1e5)
```

I_G_ratio_approx_prime

G Wishart normalising constant ratio approximation for prime graphs

---

Description

This function returns the approximation of the ratio of log transformed G-Wishart normalising constants I_G(beta, D) / I_G(beta, I) for connected prime graphs G. If there is no explicit formula, the approximation is used. Note that this is the same as the ratio C_G(delta, D) / C_G(delta, I) with delta = 2*beta + 2

Usage

```
I_G_ratio_approx_prime(
  graph,
  beta,
  D,
  use_nlo = FALSE,
  chordal = NULL,
  use_comp = NULL
)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
beta	A constant > 0
D	A p by p real symmetric positive definite matrix
use_nlo	Whether to use the next-order approximation (TRUE) or not (default, FALSE). It is more accurate, but slower, and overwrites use_comp (as TRUE)
chordal	Optional. An igraph object, corresponding to a chordal completion of graph
use_comp	Whether to approximate using the graph (FALSE), the complement (TRUE) or the more efficient (default NULL)

Value

A number approximating log(I_G(beta, D)) - log(I_G(beta, I))

Examples

```
set.seed(42)
p <- 5
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
beta <- 1
adj <- matrix(0, nrow = p, ncol = p)
adj[1,5] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_ratio_approx_prime(graph, beta, D)
I_G_ratio_approx_prime(graph, beta, D, use_nlo = TRUE)
# compare with MC estimates
I_G_MC(graph, beta, D, n_samp = 1e5) - I_G_MC(graph, beta, diag(p), n_samp = 1e5)
```

I_G_special *G Wishart normalising constant for special cases*

Description

This function returns the log of transformed G-Wishart normalising constant I_G(beta, I) = int_[S^p_++(G)] det(K)^beta * exp(-tr(K)) dK for connected graphs G for special cases where there is an explicit formula, including involving a low-dimensional integral.

Usage

```
I_G_special(
  graph,
  beta,
  connected = FALSE,
  prime = FALSE,
  test_6_cycle = TRUE,
  test_k_partite = TRUE,
  chordal = NULL
)
```

Arguments

graph	An igraph object, corresponding to a prime connected graph
beta	A constant > 0
connected	Whether the graph is connected, if known (default FALSE)
prime	Whether the graph is prime, if known (default FALSE)
test_6_cycle	Flag whether to test whether the graph is a 6-cycle or its complement (default TRUE)
test_k_partite	Flag whether to test whether the graph is complete k partite (default TRUE)
chordal	Optional. An igraph object, corresponding to a chordal completion of graph

Value

A number log(I_G(beta, I))

Examples

```
beta <- 1
p <- 5
adj <- matrix(0, nrow = p, ncol = p)
adj[1,5] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_special(graph, beta)
# compare with BDgraph gnorm()
I_G_BD(graph, beta, diag(p), n_samp = 1e5)
p <- 6
adj <- matrix(0, nrow = p, ncol = p)
adj[1,6] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- adj[5,6] <- 1 # 6-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_special(graph, beta)
# compare with BDgraph gnorm()
I_G_BD(graph, beta, diag(p), n_samp = 1e5)
graph <- igraph::complementer(graph) # complement of 6-cycle
I_G_special(graph, beta)
# compare with BDgraph gnorm()
I_G_BD(graph, beta, diag(p), n_samp = 1e5)
adj[upper.tri(adj)] <- 1
```

```
adj[2,3] <- adj[1,4] <- adj[5,6] <- 0 # complete graph with some edges missing
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
I_G_special(graph, beta)
# compare with BDgraph gnorm()
I_G_BD(graph, beta, diag(p), n_samp = 1e5)
```

`PD_complete` *PD-completion*

Description

This function returns the PD-completion of a matrix with respect to a graph. The entries of the given matrix corresponding to the missing edges are modified such that the inverse has zeros in the places of missing edges.

Usage

```
PD_complete(graph, D, missing_edges = NULL, tol = 1e-12)
```

Arguments

`graph`	An igraph object, corresponding to the graph
`D`	A real symmetric positive definite p by p matrix
`missing_edges`	optional argument of edges to complete
`tol`	A tolerance to stop the numerical iterations

Value

A real symmetric positive definite p by p matrix

Examples

```
set.seed(42)
p <- 5
data <- matrix(runif(10*p), ncol = p)
D <- t(data) %*% data
adj <- matrix(0, nrow = p, ncol = p)
adj[1,5] <- adj[1,2] <- adj[2,3] <- adj[3,4] <- adj[4,5] <- 1 # 5-cycle
graph <- igraph::graph_from_adjacency_matrix(adj, mode = c("max"))
(D_tilde <- PD_complete(graph, D))
(round(solve(D_tilde), 6)) # has zeros at the missing edges
```

prime_decomp *Prime Decomposition*

Description

This function returns the prime components and the minimal separators of a connected graph.

Usage

```
prime_decomp(graph)
```

Arguments

graph	An igraph object, a connected graph

Value

A list of the prime components and the separators

Examples

```
graph <- igraph::make_graph(edges = c(1,2, 2,4, 2,5, 1,3, 3,5,
  5,6, 6,7, 5,7, 4,7), n = 7, directed = FALSE)
prime_decomp(graph)
```

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